Aeronautical Eng. Department Level: 3rd Year Examiner: Dr. Mohamed Eid Time allowed: 3 hours		Semester: Autumn 2019 Final Exam: Mathematics IV Code: Math 301 Date: January 5, 2020

The Exam consists of one page | Answer all questions | No. of questions: 4 | Total Mark: 55

Question 1 (10 marks)

(a)Find the integrals: (i) $\int_0^{\infty} x^6\, e^{-2x}\, dx$ (ii) $\int_0^{\infty} \frac{1}{1+x^4}\, dx$ (iii) $\int_0^{\frac{\pi}{2}} \sqrt{\cot x}\, dx$ — 6

(b)By Laplace transformation, solve the PDE: $x\, u_x + u_t = x \sin t,\ u(x,0) = 0.$ — 4

Question 2 (15 marks)

(a)Solve the PDE: (i) $u_{xx} - u_{yy} = e^{2x+3y}$ (ii) $3u_{xy} - u_{yy} = \cos(3x - 2y)$ — 6

(b)Solve the wave equation: $u_{tt} - u_{xx} = 0,\ \ 0 < x < 1$

B.C. $u(0,t) = u(1,t) = 0$ and I.C. $u(x,0) = x,\ \ u_t(x,0) = 3$ — 5

(c)Solve the heat equation: $u_t - 9u_{xx} = 0,\ \ 0 < x < 1.$

B.C. $u(0,t) = u(1,t) = 0$ and I.C. $u(x,0) = 3 + x.$ — 4

Question 3 (15 marks)

(a)Write the table of frequency and the Pdf of the data: 2, 2, 2, 3, 3, 5, 5, 5, 6, 6, 8, 8. Also, find $\bar{x},\ \sigma$. — 3

(b)From the data: (1, 3), (2, 5), (4, 7), (5, 11), (7, 14).
Find the regression line $y = a + bx$ and $\bar{x},\ \bar{y},\ \sigma_x,\ \sigma_y$, cov(x, y), r. — 4

(c) If x is random variable with pdf $f(x) = \frac{1}{4}(x+1),\ 0 \le x \le 2$. Find the moment generating function $M_x(t)$ and from it, find $m_1,\ m_2,$ and σ. — 4

(d)If x, y are random variables with pdf $f(x,y) = \frac{1}{25}(x^2 + y),\ x = 1, 2,\ \ y = 0, 2, 3.$
Find cov(x, y). — 4

Question 4 (15 marks)

(a)From Beta distribution, show that $\sigma = \sqrt{\frac{m.n}{(m+n+1)(m+n)^2}}$. — 4

(b)If the probability of a defective item in production processing is 0.002. By the binomial distribution, find the probability that a lot of 300 items contains 1 defective. — 4

(c)If $\mu = 0.8$, $\sigma = 2$ in normal distribution. Find $P(2 \le x \le 3)$, $P(x > 3)$ where $\phi(1.1) = 0.8643,\ \phi(0.6) = 0.7257$ — 4

(d)From the Gamma distribution: $f(x) = \frac{1}{\Gamma(n)} x^{n-1} e^{-x},\ \ x, n > 0$
Find $P(x \le 3)$ and $P(x > 4)$ when n = 2. — 3

Good Luck *Dr. Mohamed Eid*

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(b) Solve the heat equation: $u_t - u_{xx} = 0, \quad 0 < x < 2.$ — 4

B.C. $u(0,t) = u(2,t) = 0$ and I.C. $u(x,0) = 3 + x.$

Question 2 (15 marks)

(a) Solve the PDE: (i) $u_{xx} - 9u_{yy} = e^{2x-3y}$ (ii) $u_{xx} + 4u_{yy} = x^2 y$ — 6

(iii) $u_{xy} - 2u_{yy} = \cos(3x - y)$ — 5

(b) Solve the wave equation: $u_{tt} - 4u_{xx} = 0, \quad 0 < x < 2$ — 4

B.C. $u(0,t) = u(2,t) = 0$ and I.C. $u(x,0) = 1, \quad u_t(x,0) = 2x$

Question 3 (15 marks)

(a) Write the table of frequency and the Pdf of the data: 1, 1, 1, 4, 4, 5, 5, 5, 7, 7, 8, 8. Also, find $\bar{x}$, σ. — 3

(b) From the data: (3, 3), (4, 5), (5, 11), (6, 15), (7, 14). — 4

Find the regression line $y = a + bx$ and $\bar{x}$, $\bar{y}$, σ_x, σ_y, cov(x, y), r.

(c) If x is random variable with pdf $f(x) = \frac{1}{6}(2x - 1),\ 0 \le x \le 3$. Find the moment generating function $M_x(t)$ and from it, find m_1, m_2, and σ. — 4

(d) If x, y are random variables with pdf $f(x,y) = \frac{3}{14}(x^2 + 2y),\ 0 \le x \le 1,\ 0 \le y \le 2.$ — 4

Find cov(x, y).

Question 4 (15 marks)

(a) In binomial distribution, prove that $\mu = np$. — 3

(b) A coin is tossed 6 times. By the binomial distribution, find the probability that the head appears 3 times. — 4

(c) If $\mu = 0.4$, $\sigma = 2$ in normal distribution. Find $P(2 \le x \le 3)$, $P(x > 3)$ where $\phi(1.1) = 0.8643,\ \phi(0.6) = 0.7257,\ \phi(0.5) = 0.695$ — 4

(d) From the Gamma distribution: $f(x) = \frac{1}{\Gamma(n)} x^{n-1} e^{-x},\ x, n > 0$ — 4

Find $P(x \le 4)$ and $P(x > 5)$ when n = 2.

Good Luck *Dr. Mohamed Eid*